

Thesis Abstract ¹

A combinatorial approach to representations of Lie groups and algebras

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First I describe the invariants and decompositions of tensor products of polynomial representations of $SL(2)$ in the terms of outerplanar graphs, i.e. graphs with the vertices $0, 1, \dots, m$, the edges of which can be drawn in the upper half-plane without intersections. Then I use wave graphs introduced here to give an analogous description for tensor invariants of $SL(n)$ and $Sp(2n)$. I use quite a different combinatorial approach to the description of the representations of unipotent groups of Lie type, through 'diagrams of representations'. This approach leads to various applications, including explicit formulas for fractional residues (i.e. invariants of some generalizations of differential forms). Conversely, these combinatorial approaches to representations allow us to use known representation-theoretical results to get the explicit formulas for the enumeration of the corresponding combinatorial objects, like walks on lattices, or the counting of some specific graphs.

¹University of Pennsylvania, 1998. Thesis advisor Alexandre Kirillov.