

**Math 277 01,
the course of Dr. Mihailovs**

Midterm 2

October 28, 1998

Name _____

Problem	1	2	3	4	5	6	7	8	9	Extra	Total
Points											

1. Find the eigenvalues and corresponding eigenvectors for the symmetric matrix $A = \begin{pmatrix} -1 & 2 \\ 2 & 2 \end{pmatrix}$.

2. With A as in Problem 1, find an orthogonal matrix B such that $B^{-1}AB$ is diagonal.

3. Solve the initial-value problem $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -x+2y \\ 2x+2y \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

4. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Write down e^{At} (just give the answer).

5. Let $A = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$. Write down the general real solution to $\dot{\mathbf{x}} = A\mathbf{x}$.

6. By introducing a new variable y , write $\ddot{x} - 2\dot{x} + x = 0$ as a system of two first order linear equations of the form $\dot{\mathbf{x}} = A\mathbf{x}$.

7. $A = \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}$. Solve the initial-value problem $\dot{\mathbf{x}} = A\mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

8. $\mathbf{0}$ is an equilibrium point for the autonomous system $\dot{x} = -\sin(x + y)$, $\dot{y} = xy + 2x - 4y$. Determine whether this equilibrium point is stable or unstable.

9. Solve the initial-value problem $y' = \frac{3x^2 \cos y + y^2 \cos x}{x^3 \sin y - 2y \sin x}$, $y(0) = 1$.